16. V. A. Kravchenko, "Convergent shock wave in a condensed medium," Preprint, ITF, AN SSSR, No. 4, Chernogolovka (1984).
17. M. V. Babykin, K. A. Baigarin, et al., "Symmetrical heating and dispersal of a metallic sphere with injection of an electronic pulse into an electromagnetic trap," Vopr. At. Nauka Tekh., Ser. T/ya Sintez, No. 1(18) (1985).
18. V. S. Imshennik, "Cumulation of convergent shock waves with allowance for dissipative processes," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1980).

STABILITY OF SYMMETRICAL COMPRESSION OF A CYLINDRICAL LINER MODELING A SYSTEM OF WIRES

A. A. Samokhin

UDC 533.952

Low-inductance multiwire devices ("arrays") [1], used as the load in pin-diodes, have made it possible to obtain plasmas characterized by high velocities (~10^7 cm/sec) and extreme parameters. This had in turn made such plasmas a promising tool in studies of powerful sources of electromagnetic radiation and dense-plasma generators [2]. The multiwire devices are also of interest for modeling the dynamics of the compression of cylindrical liners and z-pinches. The study of instabilities which disturb the synchronicity of the convergence of conductors at the center of the system is an important goal [3]. Here, we analyze the stability of the symmetrical collapse of an "array" with allowance for the mutual inductive effect of the currents and the finite ohmic resistance of the conductors. By using an asymptotic solution, the results are extended to the case of a solid liner.

Formulation of the Problem. We will examine a system of N rectilinear conductors (wires) with current. The conductors are positioned between two plane electrodes and close a circuit with the voltage source E, external inductance L_{ext}, and external resistance Ω_{ext}. It is assumed that the wires remain parallel to the z axis during motion and have transverse dimensions much smaller than the characteristic spacing. In this case, the motion of the liner reduces to the motion of point masses in the plane (x, y). We will use a Lagrangian formulation of the problem [4] to obtain the corresponding equations of motion with allowance for the changing inductance of the system. Each conductor is described by three generalized coordinates. Two of these coordinates (x, y) describe the position of the conductor, while the third coordinate Q gives the magnitude of the transmitted charge and corresponds to an "electrical" degree of freedom. The Lagrangian of the system, comprised of the kinetic energy of the wires, the energy of the magnetic field, and the energy of the external source, has the form

$$\mathscr{L} = \frac{1}{2}\sum^{\alpha} M_\alpha \dot{X}_\alpha^2 + \frac{1}{2}\sum^{\alpha\beta} L_{\alpha\beta}\dot{Q}_\alpha\dot{Q}_\beta + \frac{1}{2}L_{\text{ext}}\left(\sum^{\alpha}\dot{Q}_\alpha\right)^2 + E\sum^{\alpha} Q_\alpha; \tag{1}$$

$$\begin{aligned} L_{\alpha\beta} &= 2lc^{-2}\ln\left(R_\infty/|X_\alpha - X_\beta|\right),\ \alpha\neq\beta, \\ L_{\alpha\alpha} &= 2lc^{-2}\ln\left(R_\infty/r_\alpha\right). \end{aligned} \tag{2}$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 89-94, March-April, 1988. Original article submitted December 5, 1986.

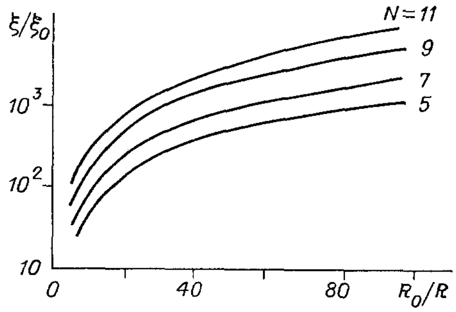

Fig. 1

Here, $X_\alpha = (x_\alpha,\ y_\alpha)$; M_α, r_α are the mass and radius of the conductor, ℓ is the length of the liner; R_∞ is the radius of the reverse conductor. If the reverse conductor is made in the form of a cylinder coaxial with the linear, then allowance for the inductive properties in the chosen form (2) is an idealization which does not consider that the portion of the conductors does not coincide with the center of the system. However, such an approximation is valid if the radius R_∞ is much greater than the radius of the linear R. In the case when the reverse conductor is made in a form similar to an array, i.e. consists of several massive conductors, then the validity of the approximation of an "effective" R_∞ needs to be investigated. This question is not addressed here. With allowance for the resistance Ω_{ext} and the resistance of the conductors Ω_α, we write the corresponding Lagrange equations as follows

$$M_\alpha \ddot{\mathbf{X}}_\alpha = I_\alpha \sum^{\beta} I_\beta \frac{\partial L_{\alpha\beta}}{\partial \mathbf{X}_\alpha}; \tag{3}$$

$$\frac{d}{dt} \sum^{\beta} (L_{\alpha\beta} + L_{\text{ext}}) I_\beta = E - \sum^{\beta} \Omega_{\alpha\beta} I_\beta,\ \Omega_{\alpha\beta} = \Omega_{\text{ext}} \quad (\alpha \neq \beta),\ \Omega_{\alpha\alpha} = \Omega_{\text{ext}} + \Omega_\alpha. \tag{4}$$

Here, $I_\alpha = Q_\alpha$ and the external voltage are equal to zero, then (4) is equivalent to the condition of trapping of the magnetic field and conservation of the generalized impulse A_z (A is the vector-potential of the magnetic field).

To achieve a high degree of compression of the linear at the initial moment, the conductors are located at the vertices of a regular polygon and it is assumed that this symmetry is maintained during the process of motion. The stability of the symmetrical compression of a multiwire linear was studied in [3]. Here, the authors ignored current fluctuations (δI_α = 0) or, equivalently, employed a regime in which the current was described. Within the framework of the model formulated above, this approximation corresponds to the case of large ohmic resistance, when the left side of Eq. (4) can be ignored and we can determine the currents from the equality of the voltage of the load to the ohmic resistance on the conductor: $I_\alpha = \left(E - \Omega_{\text{ext}} \sum^{\beta} I_\beta\right) \Big/ \Omega_\alpha$. If the resistances of the conductors are the same, then the nonsymmetrical perturbations of the current are identically equal to zero. The goal of the present study is to investigate the stability of symmetrical compression of a multiwire linear with allowance for the inductive voltage drop on the conductors.

Stability of Compression of a Multiwire Liner. We will represent the initial symmetrical solution of the problem of the compression of a multiwire liner in the form $\mathbf{X}_\alpha = \widehat{O}^\alpha \mathbf{X}$, $\mathbf{X} = R\mathbf{e}$, $I_\alpha = I$, where $\widehat{O}$ is a transformation of rotation through the angle $2\pi/N$; e = (1, 0) is the unit vector. We subject this solution to a perturbation: $\delta \mathbf{X}_\alpha = (\lambda_p \widehat{O})^\alpha \delta \mathbf{X}$, $\delta I_\alpha = \lambda_p^\alpha \delta I$, $\lambda_p = \exp(i2\pi p/N)$, $p = 0, 1, \ldots, N-1$. After linearization of Eqs. (3) and (4), if we ignore fluctuations of the transverse dimensions and resistances of the conductors we obtain

$$\left(\frac{d^2}{dt^2} + \frac{2lI^2}{Mc^2R^2}\begin{bmatrix} A_p & 0 \\ 0 & -A_p \end{bmatrix}\right) \delta \mathbf{X} = \frac{2lI}{McR}\begin{bmatrix} 1 - N\dfrac{1+\delta_{0p}}{2} \\ iB_p \end{bmatrix} \delta I; \tag{5}$$

$$\frac{d}{dt}\frac{I}{R}\left(\begin{bmatrix} 1 - N\dfrac{\delta_{0p}+1}{2} \\ -iB_p \end{bmatrix},\ \delta \mathbf{X}\right) + \\ + \frac{d}{dt}\left(C_p + \ln\frac{R}{r} + N\delta_{0p}\left[L_{\text{ext}} + \ln\frac{R_\infty}{R}\right]\right)\delta I = -\frac{c^2}{2l}(\Omega + N\delta_{0p}\Omega_{\text{ext}})\,\delta I. \tag{6}$$

TABLE 1

p	$\tilde{\gamma}$					
	N					
	4	5	6	7	8	9
2	0,71	1,00 0,96	1,22 1,11	1,41 1,20	1,58 1,27	1,73 1,32
3	—	—	1,41 —	1,73 1,71	2,0 1,91	2,24 2,07
4	—	—	—	—	2,12 —	2,45 2,43

Note: The top and bottom numbers correspond to R/r = ∞ and 10.

Here, δ_{0p} is the Kronecker symbol; the constants A_p, B_p, and C_p are determined by the equalities

$$A_p = \sum_{\beta=1}^{N-1} \frac{\cos(2\pi\beta/N) - \cos(2\pi p\beta/N)}{2(1-\cos(2\pi\beta/N))},$$

$$B_p = \sum_{\beta=1}^{N-1} \cos^2\left(\frac{\pi\beta}{N}\right) \frac{\sin(2\pi p\beta/N)}{\sin(2\pi\beta/N)}, \quad C_p = \sum_{\beta=1}^{N-1} \cos\left(\frac{2\pi p}{N}\beta\right) \ln \frac{0.5}{1-\cos\left(\frac{2\pi\beta}{N}\right)}. \tag{7}$$

The expressions presented above for A_p and B_p can be simplified. It is easy to establish the recursion formulas $A_{p+1} = A_p + B_p + 0.5(N\delta_{0p} - 1)$, $B_{p+1} = B_p - 1 + 0.5(\delta_{0p} + \delta_{pN-1})$. From here, taking into account the equalities $A_1 = 0$, $B_0 = 0$, we find

$$A_p = 0.5(p-1)(N-p-1), \; B_p = 0.5N - p, \tag{8}$$

where p takes values of 1, 2, ..., N − 1. We will henceforth limit ourselves to examination of the harmonics which disturb the symmetric of the liner, i.e. p ≠ 0. Here, the parameters of the external radius of the circuit − inductance L_{ext}, resistance Ω_{ext}, and the radius of the reverse conductor R_∞ − are omitted from Eq. (6).

We will examine the case of an ideally conducting liner (Ω = 0). For convenience, we introduce the notation $G_p = C_p + \ln(R/r)$. Using Eq. (6), we express the current perturbation δI through δX

$$\delta I = G_p^{-1} \frac{I}{R} \left(\begin{bmatrix} B_1 \\ iB_p \end{bmatrix}, \delta \mathbf{X} \right) \tag{9}$$

and, having inserted this into (5), we obtain

$$\left(\frac{d^2}{dt^2} + \frac{2lI^2}{Mc^2R^2} \left\{ \begin{bmatrix} A_p & 0 \\ 0 & -A_p \end{bmatrix} + G_p^{-1} \begin{bmatrix} B_1^2 & iB_1B_p \\ -iB_1B_p & B_p^2 \end{bmatrix} \right\} \right) \delta \mathbf{X} = 0. \tag{10}$$

To solve this equation, we ignore the change in G_p during compression of the liner and we put

$$\delta \mathbf{X} = \delta \mathbf{X}_0 \exp\left(\int_0^t \gamma(t')\,dt' \right). \tag{11}$$

In the quasiclassical approximation, γ is determined as the root of the dispersion relation

$$G_p\tilde{\gamma}^4 + (B_1^2 + B_p^2)\tilde{\gamma}^2 - A_p(G_p + 2) = 0, \quad \gamma = \tilde{\gamma}\left(\frac{2lI^2}{Mc^2R^2}\right)^{1/2}. \tag{12}$$

Considering (8), we have

$$\tilde{\gamma}^2 = \frac{(p-1)^2 + (N-p-1)^2}{4G_p} \left(\pm \sqrt{1 + G_p(G_p+2)\left[\frac{2(p-1)(N-p-1)}{(p-1)^2+(N-p-1)^2}\right]^2} - 1 \right). \tag{13}$$

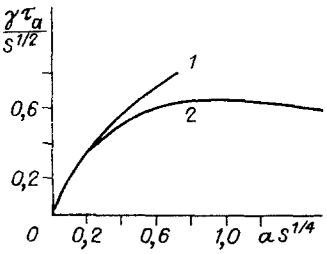

Fig. 2

It follows from Eq. (9) that the prescribed-current regime examined in [3] is realized in the limit $G_p = \infty$. Large values of G_p are attained as a result of an increase in the ratio R/r, when the mutual inductive effect of the conductors approaches zero. In this case, the tangential deformations are unstable, with an increment equal to

$$\tilde{\gamma}_\infty = \sqrt{\frac{(p-1)(N-p-1)}{2}}. \quad (14)$$

A reduction in the radius of the liner is accompanied by an increase in the inductive effect of the conductors and a corresponding reduction in the instability increments.

The effect of inductance on the instability increment is greater, the larger the value of N and the smaller the value of p. Inductance has no effect on the increment for the smallest shortwave perturbations p = 0.5N (N even).

Let us account for the resistance of the conductors. As before, we seek a solution in the form (11), we use (6) to express current fluctuations δI through $\delta \mathbf{X}$, and we obtain Eq. (9) with the factor

$$G_p = C_p + \ln\frac{R}{r} + [\tau_\Omega(\gamma + \tau^{-1})]^{-1},\ \tau_\Omega^{-1} = \Omega c^2/2l,\ \tau^{-1} = \frac{d}{dt}\ln\frac{I}{R}. \quad (15)$$

With allowance for Eq. (15), dispersion relation (12) has five roots. The value of G_p and the increment $\gamma > 0$, corresponding the unstable mode, are found by successive iterations of system (13), (15). For the unstable mode, allowing for ohmic resistance leads to an increase in the factor G_p and approach of the increment to the limiting maximum value $\tilde{\gamma}_\infty$ (14). The following conclusion can be made: for a small number of conductors N = 4-9 and $R/r \geq 10$ (see Table 1), the increments of the longwave perturbations differ from the limiting values by no more than 30% and are equal to the latter for the shortest-wave and most rapidly-growing disturbances.

We will find the increase in the amplitude of a perturbation during compression of the liner. With the condition that the current change slowly, we use the equation of motion $\frac{d^2R}{dt^2} = -\frac{2lI^2}{Mc^2R}\frac{N-1}{2}$ to obtain $\frac{dR}{dt} = -\left(\frac{2lI^2}{Mc^2}\right)^{1/2}\sqrt{(N-1)\ln\frac{R_0}{R}}$. From this and (12), if we ignore $\tilde{\gamma}$ we find the integral increment as a function of the radius

$$\Gamma = \int_0^t \gamma(t')\,dt' = \frac{2\tilde{\gamma}}{(N-1)^{1/2}}\sqrt{\ln\frac{R_0}{R}} \quad (16)$$

(R_0 is the initial radius of the liner). The relative deformation $\xi = |\delta\mathbf{X}|/R$ for rapidly growing short-wave perturbations is determined by the following relation as a function of the degree of compression of the linear (R_0/R)

$$\frac{\xi}{\xi_0} = \frac{R_0}{R}\exp\left\{\frac{2^{1/2}(0.5N-1)}{(N-1)^{1/2}}\sqrt{\ln\frac{R_0}{R}}\right\}$$

and is shown for different N in Fig. 1, where $\xi_0 = \xi|_{t=0}$.

Stability of Compression of a Thin-Walled Cylindrical Liner. Let us extend the results of the previous section to the case $N = \infty$ with prescribed finite values of current density, $j = NI/2\pi R$ and mass $m = NM/2\pi R$ per unit length and with a liner thickness $a = Nr^2/2R$. The results obtained here will be valid for perturbations with a wavelength $\lambda = 2\pi R/p \gg a$ – the longwave limit. Under the condition $N \gg P$, Eqs. (7) and (8) lead to the equalities A_pN^{-1}

$= 0.5(p-1)$, $B_pN^{-1} = 0.5$, $C_pN^{-1} = 0.5/p$. In the limit $N \to \infty$, Eqs. (10) and (15) lead to the following for perturbations $\delta X \sim \exp(\gamma t + ip\varphi)$ (φ is the polar angle)

$$\left(\gamma^2 + kg\begin{bmatrix} G^{-1}+1-p^{-1} & iG^{-1} \\ -iG^{-1} & G^{-1}-1+p^{-1}\end{bmatrix}\right)\delta \mathbf{X} = 0; \quad (17)$$

$$G = 1 + \frac{2\alpha}{\tau_a(\gamma+\tau^{-1})}. \quad (18)$$

Here, $g = 2\pi lj^2/mc^2$ is acceleration; $k = 2\pi/\lambda = p/R$ is the wave number; $\alpha = ak$; $\tau_a = 4\pi\sigma a^2/c^2$ is the time of diffusion of the field over the thickness of the linear. It should be noted that the geometry of the problem degenerates from cylindrical to planar at $p = \infty$. For the increment of the unstable mode, we have the equation $\tilde{\gamma}^2 = \frac{\gamma^2}{k\zeta} = \frac{\left\{1+\left(\frac{p-1}{p}G\right)^2\right\}^{1/2}-1}{G}$.

Let us analyze two extreme cases – strong and weak conduction. For an ideally conducting liner ($\alpha \ll \gamma\tau_a$, $G = 1$), we arrive at a Rayleigh–Taylor instability with the increment $\tilde{\gamma}|_{\sigma=\infty} = \left\{\sqrt{1+\left(\frac{p-1}{p}\right)^2}-1\right\}^{1/2} \xrightarrow[p\to\infty]{} (\sqrt{2}-1)^{1/2}$. Here, the deformation of the liner takes place simultaneously in the radial and tangential directions, with the ratio of the amplitudes $\frac{\delta R}{R\delta\varphi} = \left\{1-\frac{1}{p}+\sqrt{1+\left(\frac{p-1}{p}\right)^2}\right\}^{-1} \xrightarrow[p\to\infty]{} 0.41$. In the opposite limiting case of weak conduction, when $\alpha \gg \gamma\tau_a$, the increment increases by a factor of 1.5 while maintaining the basic dependence on kg: $\tilde{\gamma}|_{\sigma=0} = \left(\frac{p-1}{p}\right)^{1/2} \xrightarrow[p\to\infty]{} 1$. However, the relative magnitude of the radial perturbation vanishes, and deformation occurs only in the tangential direction. Such instability leads to redistribution of the substance and the formation of pinches on the surface of the liner, making it possible to classify it as a discontinuous tie-ring instability [5]. It is usual that in the present formulation of the problem, the tie-ring increment turns out to be independent of the resistance of the plasma.

Plane Current Layer. Tie-Ring Instability. Let us examine a plane current layer $j = j_z$ located in the plane (y, z) and accelerated by an external field $H = H_y$. It is modeled by a discrete grid of massive rectilinear conductors. Taking (3) and (4) as a basis and additionally considering the vector-potential of the external field, in the limit $N = \infty$ – here N is the number of conductors per unit length – we obtain the following linear equation for the perturbations $\delta \mathbf{X} = (\delta x, \delta y) \sim \exp(\gamma t + iky)$

$$\left\{\gamma^2 + kg\left(\begin{bmatrix} 1 & 0 \\ 0 & -1\end{bmatrix} + G^{-1}\begin{bmatrix} \varkappa^2 & i\varkappa \\ -i\varkappa & 1\end{bmatrix}\right)\right\}\delta \mathbf{X} = 0. \quad (19)$$

Here, as for the cylindrical liner, G is determined by Eq. (18); $g = 2\pi lj^2/mc^2$, if the external field is equal to the intrinsic magnetic field $H_m = 2\pi j/c$; $\varkappa = H/H_m$.

If $H = H_m$ and the total field on one side of the current layer vanishes, we arrive at the case of a cylindrical liner in the shortwave limit $\lambda \ll R$ ($p \gg 1$, Eq. (17)). Let us examine the stability of a massive current layer without an external field: $\varkappa = 0$. Here, the transverse ($\delta \mathbf{X} \perp \mathbf{k}$) and longitudinal ($\delta \mathbf{X} \parallel \mathbf{k}$) deformations become independent. The transverse perturbations are stable and fluctuate with the frequency $(kg)^{1/2}$. The longitudinal perturbations are unstable and, in accordance with (18), (19), the increment is determined from the equation $\gamma^2 = \frac{kg}{1+\gamma\tau_a/2\alpha}$, where $\tau_a = 4\pi\sigma a^2/c^2$; $\alpha = ak$. In conformity with the original model – which does not consider the motion of the plasma inside the layer along the transverse dimension – the condition $\alpha \ll 1$ should be satisfied. Following [5], we introduce the dimensionless number $S = \tau_a/t_a$, $t_a = \sqrt{2a/g}$ – the time of propagation of an Alfven wave through the thickness of the layer. In the limit of low conductivity, when $S \ll \alpha^{1/2}$ the rate of growth of the perturbation is determined by the hydrodynamic time $\gamma = (kg)^{1/2} = \sqrt{2\alpha}/t_a$. This result agrees with the result obtained in [5]. In the opposite limiting case of high conductivity, when $S \gg \alpha^{1/2}$

$$\gamma = \left(\frac{\Omega g}{2\pi} k^2 c^2\right)^{1/3} = \frac{(\alpha S)^{2/3}}{\tau_a}, \quad (20)$$

where $\Omega = 1/\sigma a$ is the resistance of the current layer per unit length. If we insert the value $\alpha = S^{-1/4}$ into (20) – this being the value at which the maximum increment is obtained, then we obtain the correct relation $\tau_a \gamma_{\max} \sim S^{1/2}$ [5]. There is also quantitative agreement with the results of numerical calculations in the longwave region $\alpha < S^{-1/4}$ (Fig. 2, 1 – (20), 2 –calculation from [5]).

LITERATURE CITED

1. C. Stallings, K. Nielsen, and R. Shneider, "Multiple-wire array-load for high-power pulsed generators," Appl. Phys. Lett., 29, 404 (1976).
2. G. V. Ivanenkov and A. I. Samokhin, "Nanosecond explosion of wires in the vacuum diode of a high-power accelerator," Preprint, FIAN, No. 80, Moscow (1984).
3. F. S. Felber and N. Rostoker, "Kink and displacement instabilities in imploding wire arrays," Phys. Fluids, 24, 1049 (1981).
4. V. N. Dokuka and A. A. Samokhin, "Problem on two-dimensional deformations of a thin-walled liner modeling a system of wires," Preprint, IAE, No. 4081/16, Moscow (1985).
5. H. P. Furth, J. Killen, and M. N. Rosenbluth, "Finite-resistive instabilities of a sheet pinch," Phys. Fluids, 6, 459 (1963).

EVOLUTION OF AN INTENSE SPHERICAL SHOCK WAVE IN AN INHOMOGENEOUS ATMOSPHERE

V. A. Pavlov

UDC 534.222.2

After the publication of [1] special interest arose in the propagation of intense spherical [2-6] and planar [3, 7, 8] shock waves in an inhomogeneous atmosphere. The "geometric dynamics" of intense shock waves has been developed [9, 10], and a "characteristic rule" has been proposed for such waves [10, 11]. The technique of [9-11] has been applied successfully to planar waves [10].

Relying on the concepts of [9-11] the present study will examine evolution of an intense spherical shock wave in an inhomogeneous atmosphere. The accuracy of the approximate analytical expressions obtained proves to be higher than the analogous results of [1-5].

The undisturbed state of atmospheric density and pressure are characterized by the expression $\rho_0(z)/\rho_0(0) = p_0(z)/p_0(0) = \exp(-z/H)$ (where H is the height of the "homogeneous" atmosphere). We will describe the medium by a system of gas dynamics equations

$$\partial\rho/\partial t + \mathrm{div}(\rho\mathbf{v}) = Q_1, \; d v/dt = -(1/\rho)\nabla p + \mathbf{g} = \mathbf{Q}_2, \\ dp/dt - a^2 d\rho/dt = Q_3, \quad (1)$$

where $\mathbf{v}$, $\mathbf{g}$, a are gas velocity, the acceleration of gravity, and the speed of sound; $Q_1(t, \mathbf{r}) = Q_{01}(t)\delta(\mathbf{r})$; $\mathbf{Q}_2(t, \mathbf{r}) = \mathbf{Q}_{02}(t)\delta(\mathbf{r}) || \mathbf{z}$; $Q_3(t, \mathbf{r}) = Q_{03}(t)\delta(\mathbf{r})$ is a function describing a point source. This source is located at the point R = 0 (where R, θ are spherical coordinates, z = R cos θ) and excites an intense shock wave which departs to infinity. Since the properties of the medium depend only on the single coordinate z, the source is a point, and its impulse is oriented along the z axis, the solution of Eq. (1) will have axial symmetry. It is known [5, 6] that in a wave moving upward the velocity of front displacement changes nonmonotonically, passing through a minimum at R = 1.5H/cos θ [6] (the analytical calculations of [5] give a

Leningrad. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 95-99, March-April, 1988. Original article submitted December 22, 1986.

0021-8944/88/2902-0248$12.50